1. The force, $\vec{F}$, illustrated in the figure shown below has a magnitude of 100 N.
   a) Express the force, $\vec{F}$, in Cartesian vector notation.
   b) Find the __magnitude__ of the component of vector $\vec{F}$ that lies in the direction of the displacement vector, $\vec{r}$.
   c) Write out expressions for the two vectors, $\vec{F}_{//}$ and $\vec{F}_{\perp}$. $\vec{F}_{//}$ is the vector component of $\vec{F}$ that lies in the direction of $\vec{r}$. $\vec{F}_{\perp}$ is the second vector component of $\vec{F}$ that is perpendicular to $\vec{F}_{//}$.

Solution:

Given Stated Problem

Find: $\vec{F}$, $F_{//}$, $\vec{F}_{//}$, $\vec{F}_{\perp}$.

(a) $\vec{F} = -100 \cos 30° \sin 45° \hat{i} + 100 \cos 30° \cos 45° \hat{j} + 100 \sin 30° \hat{k}$

$\vec{F} = \{-61.24 \hat{i} + 61.24 \hat{j} + 50 \hat{k}\}$ N

(b) $F_{//} = \hat{u}_r \cdot \vec{F}$; $\hat{u}_r = \dfrac{\vec{r}}{r}$

$\vec{r} = \{(-2-0)\hat{i} + (0-6)\hat{j} + 3\hat{k}\}$

$|r| = \sqrt{(-2)^2 + (-6)^2 + (3)^2} = 7.0$ m

$\hat{u}_r = -0.2857 \hat{i} - 0.8571 \hat{j} + 0.4286 \hat{k}$

$F_{//} = \hat{u}_r \cdot \vec{F}$ (scalar, dot prod.)

$F_{//} = -0.2857(-61.24) + (-0.8571)(61.24) + 0.4286(50)$

$F_{//} = -13.56$ N

(c) $\vec{F}_{//} = \hat{u}_r (\hat{u}_r \cdot \vec{F}) = -13.56(-0.2857\hat{i} - 0.8571\hat{j} + 0.4286\hat{k})$

$\vec{F}_{//} = \{3.874 \hat{i} + 11.622 \hat{j} - 5.812 \hat{k}\}$ N

$\vec{F}_{\perp} + \vec{F}_{//} = \vec{F}$; $\vec{F}_{\perp} = \vec{F} - \vec{F}_{//}$ expression

$\vec{F}_{\perp} = \{-61.24 i + 61.24 j + 50 k\} - \{3.874 \hat{i} + 11.622 \hat{j} - 5.182 \hat{k}\}$

$\vec{F}_{\perp} = \{-65.1 \hat{i} + 49.6 \hat{j} + 55.2 \hat{k}\}$ N

2a) A load Q is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. Knowing that P = 750 N, determine the tension in cable ACB and the magnitude of load Q.
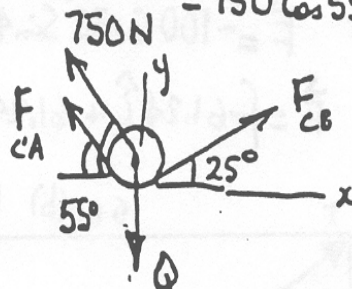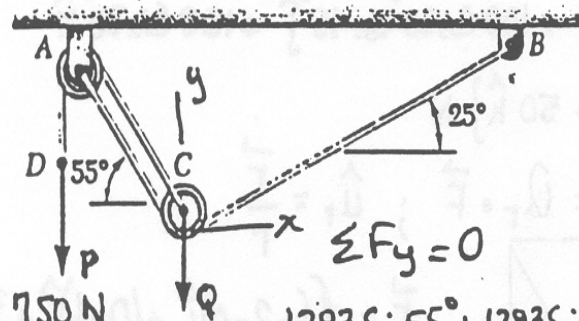
Given: Stated Problem & Figure
Find: F_{ACB} cable tension & Q.

Soln: Equilibrium Problem (2D).

$\Sigma F_x = 0 \; ; \; F_{CA} = F_{CB}$

FBD Pulley $_G$ C:

$-750 \cos 55° - F \cos 55° + F \cos 25° = 0$

$-430.2 - 0.5736 F + 0.9063 F = 0$

$F = \dfrac{430.2}{0.3327}$

$F = 1293 \text{ N}$ cable tension ACB

$\Sigma F_y = 0$

$1293 \sin 55° + 1293 \sin 25° + 750 \sin 55° - Q = 0$

$Q = \underline{2220 \text{ lb}}$ Load directed down as shown

2b) Three cables are used to tether a balloon as shown. Determine the vertical force $\vec{P}$ exerted by the balloon at A knowing that the tension in cable AB is 259 N.
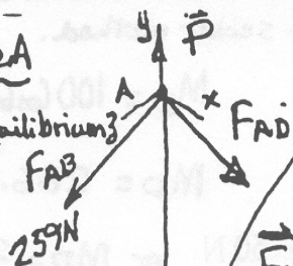
Given: Problem shown.

Find: $\vec{P_y}$.

Solution: ➝ $\vec{P}$ Pt. A is in equilibrium

FBD@A

3D Equilibrium Problem.
Write each Force in Vector format: Place in eql.
Then $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0$

$$\vec{F_{AB}} = 259 \cdot \hat{u}_{AB} = 259 \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} \text{ lb}$$

$$\vec{r}_{AB} = (-4.20 - 0)\hat{i} + (0 - 5.6)\hat{j} + 0\hat{k} \quad m$$

$$|\vec{r}_{AB}| = 7.0 \, m \longleftarrow \sqrt{(-4.2)^2 + (5.6)^2} \text{ etc.}$$

$$\vec{F_{AB}} = \left\{ 259\left[\left(\frac{-4.2}{7.0}\right)\hat{i} - \left(\frac{5.6}{7.0}\right)\hat{j}\right]\right\} N$$

$$\vec{F_{AB}} = \{-155.4\hat{i} - 207.2\hat{j} + 0\hat{k}\} N$$

$$\vec{F_{AD}} = F_{AD}\,\hat{u}_{AD} = F_{AD}\frac{(0\hat{i} - 5.6\hat{j} - 3.3\hat{k})}{6.5} N$$

$$\vec{F_{AC}} = F_{AC}\,\hat{u}_{AC} = F_{AC}\frac{(2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k})}{7.40} N$$

$$\vec{P} = P\hat{j} \qquad \sqrt{(2.4)^2 + (-5.6)^2 + (4.2)^2} = 7.40$$

$\Sigma F_x = 0$ collect $\hat{i}$'s & solve: $(\hat{i}$'s $= x$'s$)$

$$-155.4\hat{i} + F_{AC}\,0.324\hat{i} = 0; \quad F_{AC} = 479.2 \, N$$

$\Sigma F_z = 0$ collect $\hat{k}$'s & solve!

$$479.2\frac{(4.2)}{7.4}\hat{k} - F_{AD}\frac{(3.3)}{6.5}\hat{k} = 0; \quad F_{AD} = 535.7 \, N$$

ΣF_y for $\vec{P}$, collecting $\hat{j}$'s:

$$P\hat{j} - 207.2\hat{j} - 535.7\left(\frac{5.6}{6.5}\right)\hat{j} - 479.2\left(\frac{5.6}{7.4}\right)\hat{j} = 0$$

$$\vec{P} = 1031.3\hat{j}\,N \text{ or } 1031\hat{j}\,N \uparrow$$

(Diagram labels: 5.60 m, B, D, 4.20 m, O, 3.30 m, C, 4.20 m, 2.40 m, $\hat{j}$, $\hat{i}$, $\hat{k}$, A, 259N, F_{AB}, F_{AC}, F_{AD}, $\vec{P}$)

3a) The two-dimensional object experiences the three forces shown. Using scalar methods, calculate the moment at point D, located 1m from O along the x-axis.

Given: Problem shown.   Find: M_D scalar method.   2D-Moment Summation Problem. Sol'n:

$M = \Sigma F d_\perp$ $\curvearrowleft + M$ ccw
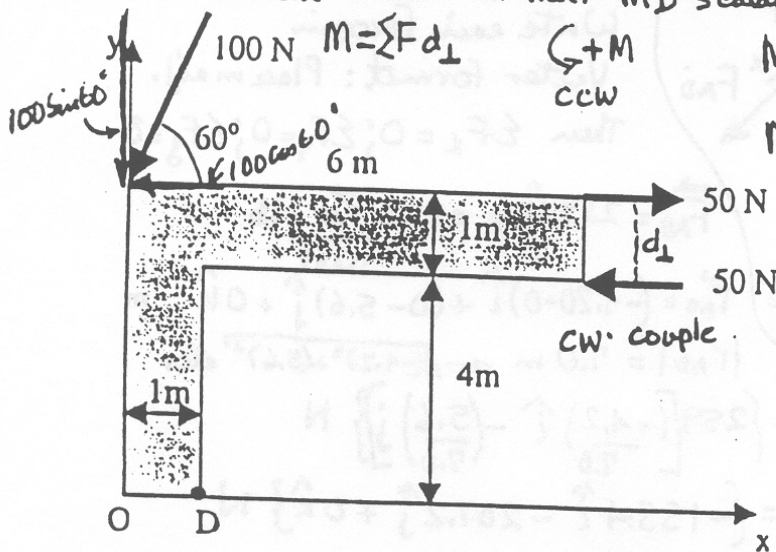
$M_D = 100\cos60°(5) + 100\sin60°(1) - 50(1)$ Nm

$M_D = 286.6$ Nm   CCW   free vector couple

or $M_D = 287$ Nm $\curvearrowleft$ ccw

3b) Calculate the moment vector $\vec{M}_{mn}$ generated by the 200 lb force about axis m-n using Cartesian vectors. Point n lies on the y-z plane, 4 ft right and 2 ft down from O.

Given: Problem shown.
Find: $\vec{M}_{mn}$

Note:
Could also use
$\vec{r}_{no}$ to cross $\vec{F}$
$\vec{r}_{no} = (0\hat{i} - 4\hat{j} + 2\hat{k})$ F_3

Soln:

$* \ \vec{M}_{mn} = \hat{u}_{mn}\left(\hat{u}_{mn} \cdot \left[\vec{r}_{mo} \times \vec{F}_3\right]\right)$; $\hat{u}_{mn} = \dfrac{\vec{r}_{mn}}{|\vec{r}_{mn}|}$

$\hat{u}_{mn} = \dfrac{(0-3)\hat{i} + (4-0)\hat{j} + (-2-0)\hat{k}}{\sqrt{(-3)^2 + (4)^2 + (-2)^{2^{7}}}} = \dfrac{-3\hat{i} + 4\hat{j} - 2\hat{k}}{5.3852}$

$\hat{u}_{mn} = \{-0.5571\hat{i} + 0.7428\hat{j} - 0.3714\hat{k}\}$

$\vec{r}_{mo} = (-3\hat{i} + 0\hat{j} + 0\hat{k})$ ft

$\vec{F}_3 = (200\hat{k})$ lb

$M_{mn} = \hat{u}_{mn} \cdot \vec{r}_{mo} \times \vec{F}_3$

$M_{mn} = \begin{vmatrix} -0.5571 & +0.7428 & -0.3714 \\ -3.00 & 0 & 0 \\ 0 & 0 & 200 \end{vmatrix}$ ft.lb

$M_{mn} = -0.7428(-3.00)200 = 445.7$ ft lb

From $*$ $\vec{M}_{mn} = (-0.5571\hat{i} + 0.7428\hat{j} - 0.3714\hat{k})445.7$ ft lb

$\therefore \vec{M}_{mn} = \{-248.3\hat{i} + 331.1\hat{j} - 165.5\hat{k}\}$ ft lb.

or $\vec{M}_{mn} = \{-248\hat{i} + 333\hat{j} - 165.5\hat{k}\}$ ft. lb. (rounded).